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Entitled

LAX INTEGRABILITY AND SOLUTION METHODS FOR THE NONLINEAR SCHRÖDINGER EQUATION WITH EXTERNAL POTENTIALS: EXACT SOLUTIONS AND APPLICATIONS

by

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Abstract

The investigation of a nonlinear differential equation (NLDE) integrability is the foundation for understanding and predicting the behavior of systems that are governed by that NLDE. The existence of a Lax pair (LP) representation is a reliable indicator of the integrability of a NLDE. If a NLDE admits a LP, it is considered integrable in the LP sense, and this has important implications for the behavior and solutions of the system. The LP plays a crucial role in many powerful analytical solution methods for solving NLDEs. One such method is the Darboux transformation (DT), which allows for the derivation of a series of exact solutions. By using the LP representation, the DT can construct a new solution from a known solution, allowing for the systematic generation of families of solutions to the NLDE. The nonlinear Schrödinger equation (NLSE) with external potentials describes many important physical systems. In the absence of external potentials, the NLSE is completely integrable, and a large number of its exact solutions are known. The presence of the potential renders the NLSE generally to a nonintegrable differential equation. Given the importance of the equation at hand, in this dissertation we consider three main streams. The first stream focuses on the analytical investigation of the NLSE's integrability in the sense of the existence of LP and the search for new exact solutions, using existing, modified, and new solution methods. The second stream focuses on the development of a high accuracy numerical method based on an iterative power series approach, capable of providing long-time evolution of solutions. The third stream involves numerical simulations to investigate the scattering of different solitons by reflectionless potentials, with emphasis on finding the spectrum of bound states. The results of this dissertation are significant contributions to the efforts made towards the NLDE integrability, exact solutions, soliton scatterings, and applications. Our study yielded the following results: *1.* Development of a method for calculating the Lax pairs spectral system for different variations of the NLSE. *2.* Calculating new Lax pairs f or different variations of the NLSE. *3.* Calculating the complete spectral system for the dual NLSE. *4.* Obtaining exact solutions from known solutions by using the DT method to the dual NLSE. *5.* Establishing a new superposition principle for the *N*-coupled NLSE. *6.* Development of a high accuracy power series method for solving partial differential equations with emphasis on the NLSEs. *7.* Establishing the necessary conditions for achieving soliton ejection and the controllability on the ejection speed value in terms of the potential and incident soliton parameters. *8.* Proving the quantum reflection of dark solitons in two setups, in the presence of external potentials and in the presence of an *x*-dependent dispersion. *9.* Calculating the critical speed for quantum reflection of the NLSE dark soliton by a reflectionless potential barrier. *10.* Identifying a class of potentials for which the scattering of flat-top solitons and thin-top solitons of the NLSE with dual nonlinearity can be reflectionless. *11.* Proving the resonant scattering and quantum reflection for the flat-top and thin-top solitons in a similar manner as for the bright soliton scattering by a reflectionless Pöschl-Teller potential. *12.* Obtaining the structure of bound-states spectrum for the Pöschl-Teller and square potential wells in the NLSE. We believe this dissertation is a significant contribution to the efforts mad towards the NLDE integrability, exact solutions, soliton scatterings, and applications.

Keywords: Integrability; Lax pair; Darboux transformation; Nonlinear Schrödinger equation; Exact solutions; Soliton-soliton interaction; Soliton-potential scattering; Reflectionless potential; Pöschl-Teller potential.