



The College of Graduate Studies and the College of Science Cordially Invite You to a
PhD Dissertation Defense

Entitled

ON THE PROJECTIONS AND UNITARY GROUPS OF UNITAL C-ALGEBRAS*

by

Fouzia Shaheen

Faculty Advisor

Prof. Ahmed Al Rawashdeh, Department of Mathematics

College of Science

Tuesday, 16 April 2024

10:00 am

Room 036, F3 Building

Abstract

H. Dye proved that the unitary group in a factor determines the algebraic type of that factor. Al-Rawashdeh, Booth and Giordano established that, for a large class of simple unital C*-algebras, an isomorphism between the unitary groups, induces an isomorphism between their K_0 -ordered group and K_1 -groups. Then using the results of Dadarlat-Elliott-Gong and Kirchberg-Phillips, the C*-algebras are isomorphic. Dye introduced special projections $P_{\{i,j\}}(a)$ of the matrix algebra $M_n(A)$, and he used it as a main tool to establish his results in the case of von Neumann factors. Precisely, in case of von Neumann algebra, he proved that if θ is an orthoisomorphism which fixes all the $P_{\{i,j\}}(a)$, then it is the identity mapping on the projections. We discuss these projections and we give more properties in the case of C*-matrix algebras. Using Dye's approach, we prove that for a unital C*-algebra A , if θ is an orthoisomorphism on $P(M_n(A))$ which fixes the $P_{\{i,j\}}(a)$, then θ fixes all the projections on class D , consisting of some decomposition of $P_{\{i,j\}}(a)$. We introduce the invariant unitary groups property (IUG-P), the orthogonal IUG-P and the topological IUG-P. We investigate that some properties are IUG-P, orthogonal IUG-P or topological IUG-P, for certain C*-algebras. If the general linear groups $(GL(A))$ are isomorphic, we prove that the induced mapping between idempotents preserves the orthogonality, for a large class of unital C*-algebras, including certain type of UHF-algebras, 2-divisible K_0 -groups, Cuntz algebras O_n , $2 \leq n \leq \infty$, and for simple unital purely infinite C*-algebras having 2-divisible K_0 - groups. We prove that if N is a normal subgroup of $GL(O_n)$, then N contains all the symmetries of O_n . Also, we show that if N is any normal subgroup of unitary groups of compact operator K , which contains some certain type of involution, then N contains all the involutions of K .